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Reliability Analysis of Space Exploration Truss Support Structures

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An adaptive reliability estimation technique for three-dimensional trusses used in space exploration applications is proposed. Both component-level and system-level reliability issues are addressed. For individual member-level performance criteria, efficient analytical methods are implemented for reliability estimation. Multiple potential failure paths are considered for system-level performance, and a hybrid adaptive sampling technique is proposed by combining the best features of analytical and sampling methods. A significant failure path is enumerated using analytical reliability estimation and branch and bound search. An adaptive importance sampling method is used to identify other significant failure paths. The proposed methodology is illustrated for application to a simplified solar array support truss, using a commercial finite element code.

Nomenclature

a_{ij}	=	load on the ith member due to the jth unit load applied on the structure where yielding has already occurred
b_{ij}	=	load on the ith member due to the jth unit load applied on the structure
d_0	=	cluster radius
$f_X(\boldsymbol{x})$	=	probability density function of random variables $\boldsymbol{X}$
$f_x^{(j)}(\boldsymbol{x})$	=	original density function with the mean shifted to $\hat{\boldsymbol{x}}^{(i)}$
$g(\boldsymbol{X})$	=	performance function
$h_X(\boldsymbol{x})$	=	importance sampling density function
n_r	=	number of applied loads
P_f^*	=	maximum path probability
P_{fs}	=	joint probability of multiple events
P_i	=	load variable
p_f	=	probability of failure
R_j	=	tension yield capacity at the jth plastic hinge already formed
R_r	=	load capacity at the location being checked
S_i	=	set of all sample points in the failure domain identified earlier
u	=	equivalent uncorrelated standard normal variable
$\boldsymbol{X}$	=	vector of random variables
$\hat{\boldsymbol{x}}^{(i)}$	=	representative point for multimodal sampling density
α_{ir}	=	rth component of unit sensitivity vector for the ith performance function
β	=	reliability index
γ	=	failure sequence truncation parameter
δ	=	relative change in failure probability
ε_c	=	small number
ρ_{ij}	=	correlation coefficient
Φ	=	standard normal cumulative distribution function
$\hat{\omega}_i^j$	=	weight of the jth representative point

Introduction

THE use of three-dimensional trusses is common in space exploration applications, for example, in space telescope supports, space station structures, solar array supports, satellite tethers, etc. For example, the integrated truss structure (ITS) forms the backbone of the International Space Station (ISS), formed by 10 preintegrated truss segments. Each segment provides the foundation for subsystem hardware installation, utility distribution, power generation, heat rejection, space experiments platforms, external payload accommodations, and a mobile transporter for robotic assembly and maintenance operations. Laboratories, living quarters, payloads, and systems equipment will be directly or indirectly connected to the ITS.‡

U.S. solar arrays supplying 105 kW of power will be attached to the ITS. Each solar array consists of two solar cell blankets, one on either side of a telescoping mast that extends and retracts to form or fold the solar array wing. Each 108.6-ft-long solar array wing will be connected to the ISS's 310-ft-long truss and extend outward at right angles to it. When fully extended, a pair of wings and their associated equipment will span about 240 ft, the largest deployable space structures ever built.

Similarly, the Russian Science Power Platform will have eight solar array wings with a span of about 100 ft. The wings will be attached to the top of one of two segments that will form a tower 26 ft tall. The control module Zarya's solar array wings will span 72 ft, and the service module's solar array wings will span 97 ft.

It is clear from the preceding discussion that these large structures, deployed in extra-terrestrial environments, may be subjected to mechanical, thermal, and dynamic loads and performance conditions that have large variations and uncertainties. Therefore, the performance reliability and structural integrity of these structures need to be evaluated using probabilistic structural analysis methods. The reliability evaluation of the structure needs to be done both at the level of individual members and for the overall system.

Several studies have been reported on the thermal, dynamic, and progressive failure analysis of large truss structures used in spacecraft applications, and a few of these have included probabilistic concepts. Malla et al.[1,2] investigated the motion and deformation of large space structures and included thermal effects. Malla and Nalluri[3] studied the effect of member failure under dynamic loading on the overall truss performance. Pai and Chamis[4,5] considered the probabilistic analysis of a solar array panel support truss for two objectives: 1) evaluation of the scatter in structural response quantities due to variations in input variables and 2) progressive buckling probability estimation of truss members. Malla and Pai[6] also performed probabilistic analysis of a short spacer truss under joint and member imperfections.

Another application of three-dimensional trusses in space exploration is for large, tetrahedral high-precision truss platforms with diameters ranging from 10 to 15 m to support faceted reflector surfaces in several Earth science and deep space astrophysics spacecraft. Mikulas et al.[7] studied the strength, stiffness, and dynamic

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‡"International Space Station," The Boeing Company, Canoga Park, CA, URL: http://www.boeing.com/defense-space/space/spacestation [cited 15 Nov. 1999].

characteristics of such trusses. The important design performance criteria of such trusses are related to their natural frequency and displacement to ensure high-precision observations.[8] Once again, the performance of such trusses in the space environment is subject to uncertainty and requires rational probabilistic analysis to ensure performance reliability.

Reliability of engineering systems may be estimated using two approaches: testing and computation. This paper pursues the computational approach that combines probabilistic analysis with the physical computational model of the system.[9–11] For truss members, the individual performance criteria may relate to tensile and compressive strength, buckling stability, displacement, natural frequency, etc. Corresponding to each criterion, a performance function $g(\boldsymbol{X})$ is mathematically formulated such that $g < 0$ represents unsatisfactory performance (or failure), $g > 0$ represents satisfactory performance or safety, and $g = 0$ is referred to as the limit state. Here, $\boldsymbol{X}$ is the vector of random variables relating to the loads and system properties. Reliability is defined as the probability of satisfactory performance, that is, $P\{g(\boldsymbol{X}) > 0\}$, and may be estimated using either analytical methods or simulation methods. Efficient analytical methods construct first-order and second-order estimates of this probability, using optimization and probability transformation techniques. The simulation methods generate numerous samples of the random variables, evaluate the structure for each sample, and compute the failure probability as the number of failure samples divided by the total number of samples.

System-level reliability analysis addresses two types of issues: 1) multiple performance criteria, or multiple limit states (even for a single member), and 2) multiple paths or sequences of individual component failures leading to overall system failure. In the case of a single member, failure may be defined as the union of several possible failures, such as strength, stability, etc. The second issue requires progressive failure analysis of the structure (i.e., reanalysis after each individual failure to identify the next likely failure location), leading to the enumeration of important failure sequences. In such analysis, component failures may be modeled as ductile (full residual capacity after failure), brittle (no residual capacity after failure), or semibrittle (partial residual capacity after failure).

The algorithms to compute system reliability also fall into two categories, namely, analytical methods and simulation methods. Analytical methods present elegant approaches to enumerate significant failure sequences but lead to approximate bounds and restrictive simplifying assumptions on structural behavior. The simulation methods are simpler to implement, robust in performance, and can incorporate practical structural behavior, but tend to be computationally expensive for realistic high-reliability systems. Therefore, this paper proposes a hybrid approach that combines the best features of analytical and simulation methods to estimate efficiently and accurately structural system reliability. The proposed methodology is applied to a three-dimensional truss support structure useful for space exploration applications.

Individual Performance Criteria

The failure probability corresponding to an individual performance criterion is estimated through the definition of a limit state corresponding to that criterion and by integrating the joint probability density function of all of the random variables over the region of failure, as

$$p_f = \int_{g(X) \le 0} f_X(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x} \tag{1}$$

where $g(\boldsymbol{X}) \le 0$ represents the failure region, and $f_X(\boldsymbol{x})$ is the joint probability density function of the random variables $\boldsymbol{X}$. Two types of methods, analytical and sampling based, are available for evaluating the multidimensional integral in Eq. (1). In the analytical method, the failure probability is estimated through a first-order approximation to the limit state, as shown in Fig. 1. The original variables $\boldsymbol{X}$ are all transformed to equivalent uncorrelated standard normal variables u, and the closest point to the origin on the limit state is found. This minimum distance point is the most probable point (MPP) of failure on the limit state. In the first-order reliability

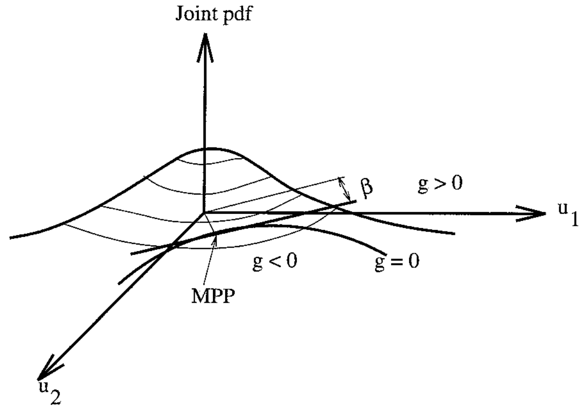

Fig. 1 First-order reliability method.

method (FORM), a first-order estimate of the failure probability is obtained as

$$p_f = \Phi(-\beta) \tag{2}$$

where β, the reliability index, is the distance from the origin to MPP and Φ is the cumulative distribution function of a standard normal variable. Second-order estimates of the failure probability by making use of the curvature of the limit state at the MPP have also been derived.[12,13]

In the Monte Carlo sampling-based method, sample values of the random variables are generated according to their probability distributions, and the performance function is evaluated using structural analysis. The probability of failure is easily computed as the number of failures divided by the total number of samples. Such an approach is simple to implement but very time consuming for high-reliability structures because a large number of samples are required to obtain a few failures.

Multiple Performance Criteria

As mentioned earlier, even a single member has to satisfy multiple performance criteria such as strength, stability, stiffness, etc. Therefore, the reliability evaluation of a single truss member first requires the reliability computation corresponding to each individual criterion. Then, the overall member failure probability is defined as the probability of union of individual failures. A general formula to compute the joint probability of more than two events was provided by Gollwitzer and Rackwitz,[14] where the theory of asymptotic approximations is used. For simpler computation, first-order[15] and second-order[16,17] bounds are also available in the literature.

The alternative, simulation-based method proceeds in the same manner as mentioned in the preceding section. The joint failure probability is simply the number of joint failures divided by the total number of samples to compute $\boldsymbol{X}$. To compute the overall member probability, samples with any failure mode get counted.

Multiple Failure Sequences

For frame and truss structures with a high degree of redundancy, there can be several possible ways to reach system failure. Each such path is called a failure sequence. For large structures, there are a large number of failure sequences, and it is practically impossible to enumerate each sequence. However, in most of the cases, only a small fraction of the sequences contribute significantly to the overall failure probability of the system.

The branch and bound method, used to search for the significant sequences, involves two main operations, namely, the branching operation and the bounding operation (Fig. 2). In the branching operation, starting from an intact structure, failure is imposed at the most likely location as indicated by the reliability analysis of all of the components. The structure is reanalyzed with the imposed failure, and the next failure is imposed at the location with the highest path probability. This process is repeated until a complete failure sequence is obtained. The main purpose of the bounding operation or truncation is to discard the insignificant failure sequences by comparing their path probabilities to the system failure probability

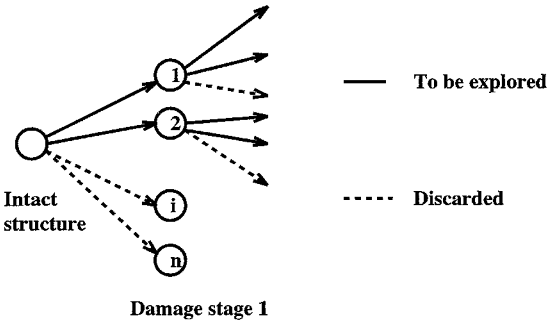

Fig. 2 Branch and bound method.

P_f. The ith failure sequence is ignored if its failure probability satisfies the following criterion:

$$P_i < \gamma P_f \tag{3}$$

where γ is parameter, with a chosen value based on the required degree of accuracy. Because P_f is unknown, it is replaced by the maximum path probability P_f^* among the significant failure sequences already identified. The accuracy and efficiency of the procedure depends on how close P_f^* is to the true system failure probability P_f.

This procedure can be quite cumbersome for large structures that may have many significant failure sequences and many component failures within a sequence. Therefore, Xiao and Mahadevan[18] developed a faster branch and bound method, where highly correlated failures are imposed in groups. Even with this improvement, the overall system failure probability has to be estimated through approximate first-order or second-order bounds because the estimation involves the union of significant failure sequences, not the union of easily defined limit states with closed-form expressions.

Adaptive Monte Carlo Simulation

Monte Carlo simulation is always a viable alternative for system reliability analysis because it provides an easy computational methodology and is robust for implementation to practical systems. However, the basic simulation method is very time consuming, as mentioned earlier. It requires a very large number of simulations to estimate the failure probability of practical, high-reliability systems. To overcome this difficulty, several efficient sampling methods and variance reduction techniques have been developed. One of the promising techniques among these is the method of importance sampling, which uses samples from the important region, the failure domain, in this case.

The system failure domain is defined as the union of failure domains defined by the significant failure sequences, and the system failure probability is the integral of the joint probability density function of the random variables over the system failure domain:

$$p_f = \int_{g_1(\boldsymbol{x}) \le 0 \,\cup\, g_2(\boldsymbol{x}) \le 0 \,\cup \cdots \cup\, g_n(\boldsymbol{x}) \le 0} f_X(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x} \tag{4}$$

where $g_1(\boldsymbol{x}), \ldots, g_n(\boldsymbol{x})$ can be considered to be the n limit states corresponding to the n significant failure sequences of the system. Here, $f_X(\boldsymbol{x})$ is the joint probability density function of all of the input random variables (both load and resistance variables) from which the sample points are generated. For importance sampling, the preceding equation can be rewritten as

$$p_f = \int_{g_1(x) \le 0 \,\cup\, g_2(x) \le 0 \,\cup \cdots \cup\, g_n(x) \le 0} \frac{f_X(\boldsymbol{x})}{h_X(\boldsymbol{x})} h_X(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x} \tag{5}$$

where $h_X(\boldsymbol{x})$ is the new sampling density function that focuses the sampling in the failure region and helps in faster convergence to the true failure probability.

For any importance sampling technique to be effective, one must have some prior knowledge of the sampling domain, that is, the system failure domain. Several methods have been proposed for the selection and adaptive refinement of the sampling domain,[19–21] including multimodal[22,23] and curvature-based[24] methods. These methods

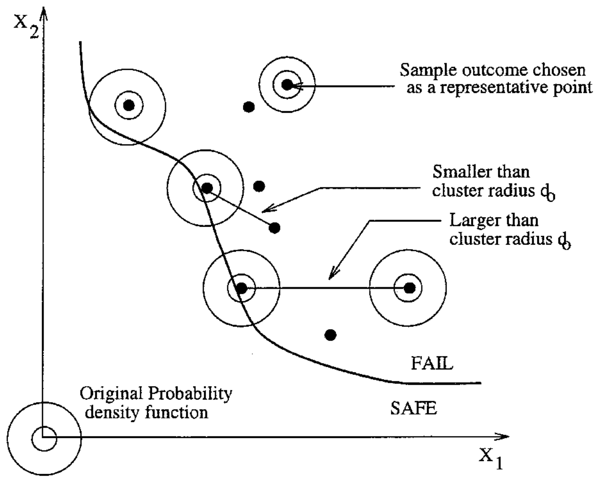

Fig. 3 Adaptive importance sampling.

have been applied mostly to single and multiple limit state problems, but not to problems with multiple failure sequences.

The importance sampling method that is used here is based on the technique developed by Karamchandani et al.[22] In this method, the initial sampling density function, is chosen to have the same form and variance as the original density function, but is centered around an initial starting point in the failure domain. Once several samples have been obtained in the failure domain, a multimodal sampling density function is constructed that emphasizes multiple points in the failure domain, each in proportion to the true probability density at the point. However, not all of the sample points are emphasized; only one representative point from a cluster of points is chosen. The representative points are separated by a distance greater than the cluster radius d_0 (see Fig. 3). Usually the value of d_0 is taken to be the average distance between the mean and the sampling points. The multimodal sampling density to generate the ith sampling point is

$$h^i_{\tilde{X}}(\tilde{\boldsymbol{x}}) = \sum_{j=1}^{k} \hat{\omega}^j_i f^{(j)}_{\tilde{X}}(\tilde{\boldsymbol{x}}) \tag{6}$$

where $\hat{\omega}^j_i$, the weight attached to the jth representative point, is computed as

$$\hat{\omega}^j_i = \frac{p_{f\,|\,\tilde{X}=\hat{x}^{(j)}} f_{\tilde{X}}(\hat{\boldsymbol{x}}^{(j)})}{\sum_{r=1}^{k} p_{f\,|\,\tilde{X}=\hat{x}^{(j)}} f_{\tilde{X}}(\hat{\boldsymbol{x}}^{(r)})} \tag{7}$$

and $f_X(\boldsymbol{x})$ is the original density function, $f^{(j)}_X(\boldsymbol{x})$ the original density function with the mean shifted to $\hat{\boldsymbol{x}}^{(j)}$, and $\hat{\boldsymbol{x}}^{(1)}, \ldots, \hat{\boldsymbol{x}}^{(k)}$ the representative points.

The representative points are identified as follows. Let S_i be the set of all sample points in the failure domain identified earlier. A cluster radius d_0 is selected. The point with the largest probability density in S_j is selected and is called $\hat{\boldsymbol{x}}^{(1)}$. In S_i, all of the points within a radius d_0 of $\hat{\boldsymbol{x}}^{(1)}$ are eliminated. Among the remaining samples in S_i, the sample point with the largest probability density is selected and called $\hat{\boldsymbol{x}}^{(2)}$. All of the points within a radius d_0 of $\hat{\boldsymbol{x}}^{(2)}$ are eliminated. This process is repeated until there are no more points left in S_i. After i sample points, the estimated failure probability is given by

$$p_f = \frac{1}{i} \sum_{j=1}^{i} \frac{p_{f\,|\,\tilde{X}=\tilde{x}^{(j)}} f_{\tilde{X}}(\boldsymbol{x}^{(j)})}{h^j_X(\boldsymbol{x}^{(j)})} \tag{8}$$

Hybrid Approach for System Reliability Analysis

The preceding method has been studied for problems with single and multiple limit states, but not for problems with multiple failure sequences. Each failure sequence consists of multiple limit states, and a new strategy for the initiation and refinement of the important sampling density for such problems needs to be developed. This paper solves this problem in a hybrid manner. In the proposed method, the branch and bound technique is first used to determine one complete failure sequence. For a single limit state problem, adaptive importance sampling needs a starting point, that is, a location for the mean of the sampling density. The starting point is usually chosen to be the most probable point (generally identified using FORM). Similarly, in the case of system failure, this paper uses the first failure sequence to define the initial sampling domain.

Once the first failure sequence is identified using the branch and bound method, an initial sampling is done to generate a few samples (about 10 samples) that have the same system failure sequence as the first failure sequence identified by the branch and bound method. (This initial sampling is done quite easily. In each simulation, the random variables are sampled from their distributions and progressive failure analysis is carried out until system failure. If the failure sequence is the same as the first failure sequence, the sample point is accepted, otherwise, it is rejected.) From these initial samples, representative points are selected as mentioned earlier, and a multimodal sampling density function is constructed as in Eq. (6).

Adaptive importance sampling is done next. Samples are obtained with the multimodal density function, and all samples that lead to structural failure by any sequence are accepted. After each sampling, the set of representative points and, therefore, the multimodal sampling density, are modified to include the new system failure samples, thus refining the failure domain approximation. These representative points are weighted according to their actual probability density. The weighted multimodal density serves two purposes: 1) More system failure sequences in addition to the first one are accounted for, and 2) the sequences are weighted according to their probability. Therefore, if the initial failure sequence did not have high probability, or during the adaptive sampling some of the less probable sequences got included in the set of representative points, their effects would be minimized due to the smaller weight attached to such sequences.

Sampling is continued until the failure probability converges to an accepted level of accuracy. Two convergence criteria are used in this paper. The first criterion measures the stability of the estimate. The relative change in failure probability estimate with each additional sample is computed as

$$\delta = |p_{f_k} - p_{f_{k-1}}|/p_{f_{k-1}} \tag{9}$$

If $\delta \leq \varepsilon$ for 10 successive simulations, where ε is a small number, then the estimate is considered stable. The second criterion measures the scatter considering all of the samples. If the coefficient of variation (COV) of the failure probability after satisfying the first criterion is less than ε_c (a small number), then the simulation is stopped and the converged estimate of p_f is reported.

The proposed methods to estimate component and system failure probability are achieved and have been implemented by the authors through a combination of C language programming and the commercial structural analysis code ANSYS. The tasks can be divided into several distinct sections: 1) branch and bound method, to identify the first failure sequence; 2) adaptive importance sampling, to generate the samples; 3) reliability methods (FORM, SORM, and simulation based); and 4) structural analysis.

The first three sections make use of the fourth section, which performs the structural analysis using ANSYS. Except for the fourth section, all other sections are programmed in C language and linked to ANSYS.

The first section consists of tasks related to the identification of the first failure sequence, as follows:

1) Perform FORM analysis to compute the component probabilities of failure used in the branch and bound search.

2) Keep track of failures imposed on various structural components during the search for the first failure sequence.

3) Modify the structure at each stage of the branch and bound search to represent the imposed component failure.

4) Call the structural analysis program.

5) Process the output of the structural analysis program, and feed to the FORM analysis to estimate the failure probabilities of various components.

The second section consists of tasks related to the adaptive sampling, as follows:

1) Compute the cluster radius.

2) Determine the representative points.

3) Generate samples using the multimodal sampling density function.

4) Call the structural analysis program.

5) Process the results from the structural analysis program.

6) Determine the failure loads.

7) Compute conditional failure probability for the loads.

8) Compute the system failure probability at the end of each simulation.

9) Check for convergence in the system failure probability estimate, and either stop or continue the adaptive importance sampling by repeating steps 2–9.

The third section consists of tasks related to structural analysis. This section has functions that make calls to the structural analysis software (ANSYS, in this paper). The results of the analysis are passed to the adaptive sampling and the branch and bound routines for processing.

The proposed method is general enough to include all three types of structural failure: brittle, ductile, and semiductile. In the case of ductile failure (e.g., yielding of the truss member due to tension), the member has residual load capacity after failure. In the case of brittle failure (e.g., buckling under compressive load), the member does not have any residual capacity corresponding to that failure mode. In the case of semiductile (or semibrittle) failure, the member has a fraction of the original load capacity. Only the reanalysis of the structure after imposing individual failures is different in each case. All other computations such as failure sequence search, adaptive importance sampling, and component and system failure probability estimation are the same with regard to all three types of failures.

In the case of ductile failure such as yielding, once the truss member has yielded due to tension at a particular location, further analysis is carried out after imposing a plastic hinge with an axial force equal to the axial load capacity of the member. In the case of brittle failure such as buckling, once the member has failed, the member is removed from the structural model. No load is imposed on the structure because the failed member has no residual capacity. Then the structure is reanalyzed for further failures. In the case of semiductile failure, an appropriate value of the load equal to the residual capacity is imposed at the failure location, and the truss structure is reanalyzed.

Once the first failure sequence is identified by the branch and bound method, the initial sampling density to start the importance sampling is constructed according to the steps enumerated here:

1) Generate the set of resistance variables R_i. This fixes the load carrying capacity of the structure.

2) Apply unit loads on the structure corresponding to the applied loads, perform linear elastic analysis, and compute the stress resultants at each potential failure location, for example, the ends of the members. One can form limit state equations for each potential location as follows:

$$R_r + \sum_{j=1}^{n_r} a_{ij} R_j - \sum_{i=1}^{n_p} b_{ij} P_i = 0 \tag{10}$$

where, R_r is the load capacity at the location being checked for failure, R_j is tension yield capacity at the jth plastic hinge already formed, b_{ij} is the load on the ith member due to the jth unit load applied on the structure, a_{ij} is the load on the ith member due to the jth unit load applied on the structure where yielding has already occurred, n_r is the number of plastic hinges already formed, and n_p is the number of applied loads. Note that in this equation, the tension yield capacities are treated as deterministic at this stage because the resistance variable values are being sampled in step 1. The only random variables in Eq. (10) are the load variables P_i.

Note that the preceding equation is applicable to ductile, brittle, and semiductile failures. The first term refers to the load capacity in any failure mode. The second term includes residual capacities only at ductile and semiductile failure locations because there is no residual capacity after brittle failure. If there were only brittle failures, the second term would be absent.

3) A FORM analysis is performed for each of the intact locations to determine the failure load(s) and the probabilities of various types of failure (yielding, buckling, etc.).

4) At the location with the highest probability of failure, the structural model is modified depending on whether the failure is ductile, brittle, or semibrittle.

5) Steps 1–3 are repeated until a complete failure sequence is realized, that is, the structural stiffness matrix becomes singular.

6) If the failure sequence identified in steps 1–5 matches the failure sequence identified earlier by the branch and bound method, then the sample point is accepted; otherwise it is rejected.

7) After this initial sampling, a set of representative points are chosen using the cluster radius approach of Fig. 3, to emphasize the region of the initial sequence in the sampling domain. A multimodal density function is constructed using Eq. (6).

Next, adaptive importance sampling is started with this multimodal density function. As new samples are generated, the representative points and the multimodal sampling density are modified, and the sampling is continued until convergence in the system failure probability estimate.

Application Problem: Solar Array Panels Mast

Photovoltaic power is the primary power source for current inner solar system space station. Photovoltaic power systems or solar arrays utilize solar cells to harness the solar energy and convert it to electrical energy. The solar arrays are of two types: body mounted and deployable. Increasing power requirements of spacecraft are necessitating the use of deployable, lightweight solar arrays. As mentioned in the introductory section, the supporting truss structures for the deployable solar arrays are subjected to significant variations in thermal and torsion loads, causing stability and reliability concerns. This numerical example features the reliability computation of such structures using the proposed method.

A simplified cantilever space truss as shown in Fig. 4 is considered for the sake of illustration. The dimensions and properties of this cantilever space truss are taken from Pai and Chamis.[4,5] Figure 4 also shows the loading on the cantilever space truss. There are three loads acting on the space truss. Load L1 is horizontal (at six points), load L2 is directed vertically upward (at two points), and L3 is a torque (on two members). The space truss has four supports.

The space truss is modeled in ANSYS using three-dimensional pipe elements. The space truss is modeled with three-dimensional pipe elements (PIPE20) to allow torsion loading. The statistics of

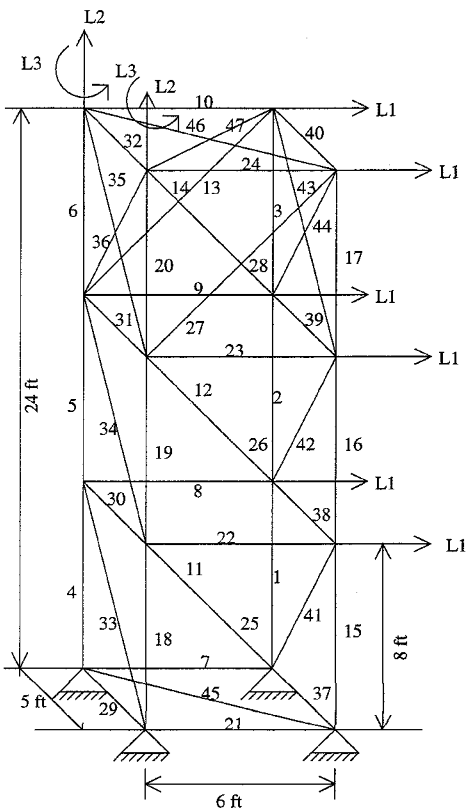

Fig. 4 Solar array mast cantilever truss: members and loading.

Table 1 Statistics of the variables for the solar array panels mast

Variable	Mean	Coefficient of variation	Distribution
Young's modulus	10^7 lb/in.^{2}	0.075	Normal
Tube radii	0.5 in. (outer) 0.44 in. (inner)	0.075	Normal
R	7087.4 lb	0.10	Lognormal
L1	600 lb	0.15	Lognormal
L2	600 lb	0.15	Lognormal
L3	1500 lb.in.	0.15	Lognormal

Table 2 Results of the solar array panels mast

Sequences	%
1-2-11-12-13-15-16-25-26-28-32	59
8-28-22-17-6-31	29
9-17-41-40-28-45	10
9-17-27-28-21-7-29	2

the random variables are given in Table 1, where R is the load capacity of the truss members (same in tension and compression). All members are assumed to have the same cross section.

A computationally efficient strategy is followed to minimize the effort in structural analysis. During the branch and bound search, linear structural analysis is used. This is because the initial sampling domain is only to get an approximate starting region. It does not have to be very accurate. Therefore, linear analysis is adequate. However, during adaptive sampling to estimate eventually the failure probability, nonlinear structural analysis is used for the sake of accuracy.

The first failure sequence is determined by using the branch and bound method. The members are assumed to fail in two modes: yielding due to tension and buckling due to compression. Equation (10) is used as the limit state for each member. A first-order reliability analysis using FORM is performed to determine the probabilities of failure of the individual members. For the branch and bound search, R_i is a random variable. The limit state with the highest probability of failure is identified. If the member fails in tension yielding (ductile failure), the member is removed for further analysis and a load equal to the load carrying capacity of the member is applied along the member. If the member fails in compression buckling (brittle failure), the member is removed for further analysis. Thus, the first significant sequence is identified for the current example as 1-2-11-12-13-15-16-25-26-28-32.

The initial sampling density is constructed with representative points among the few initial samples that have the preceding failure sequence. Next, during adaptive importance sampling, samples with other significant sequences also get included. For each simulation, the resistance variables are generated from the updated sampling density, and the failure load of the system is determined.

The ANSYS nonlinear analysis subroutine does not specify the members that have failed. Hence, additional elastic analyses were performed to determine the failure sequences based on the generated resistances and the failure loads. Thus, the number of calls to the structural analysis routine is actually double the number of simulations of adaptive sampling. However, linear elastic analysis is very quick; therefore, in terms of computational time, the effort is not doubled.

The progressive buckling of this truss has been studied earlier by Pai and Chamis,[5] using NESSUS probabilistic structural analysis software. A simplified version of the enumeration method was used, where only one failure sequence was considered. As mentioned earlier, the enumeration technique is tedious and approximate in the case of large structures with multiple potential failure sequences. The proposed hybrid method includes the significant failure sequences through adaptive sampling and provides an accurate estimate of the overall system reliability.

The significant sequences and their relative contributions are shown in Table 2. The first sequence was used for initial sampling, and it is seen that as the sampling progresses, other sequences are also included. For adaptive sampling, two convergence criteria were

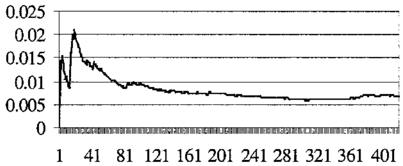

Fig. 5 Variation of failure probability with number of samples.

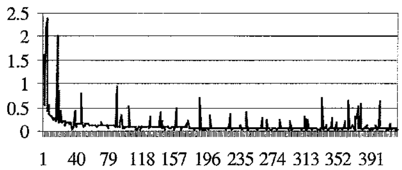

Fig. 6 Variation of COV with number of samples.

used: 1) The COV of the failure probability estimate should be less than 0.05. 2) The relative change in system failure probability estimate [computed using Eq. (8)] during the last 10 simulations should be less than 0.01. With these criteria, the proposed adaptive sampling technique took a total of 423 samples to converge to a failure probability of 0.0086. Basic Monte Carlo simulation (10,000 samples) for this problem results in a failure probability estimate of 0.0083. The results from the adaptive simulation technique closely agree with the results from basic Monte Carlo simulation. Thus, the adaptive simulation technique is more economical without sacrificing accuracy.

Figures 5 and 6 show the convergence characteristics of the adaptive sampling technique. It is seen from Fig. 5 that the failure probability p_f reaches a stable value after about 250 simulations. However, the COV of p_f continues to have frequent sharp spikes, although it decreases with the number of samples. This behavior is typical of importance sampling, where one random sample could disturb the COV. This is the reason for the first requirement that p_f be stable for several iterations.

Note that these results are specific to the assumed distributions of the random variables. Other distributions may be appropriate for different structures and applications. The choice of distribution and COV of a significant random variable may significantly affect the results in two ways: 1) The overall system failure probability estimate may change. 2) Different significant failure sequences may be identified. For example, a larger COV may result in the identification of more significant failure sequences.

Conclusions

An adaptive reliability analysis methodology is proposed in this paper for application to structural systems. The proposed method combines both analytical and simulation-based methods for efficient component and system reliability estimation. The proposed technique is demonstrated for application to a three-dimensional truss used in a solar array support structure. The truss members are subjected to axial loads and torsion moments and are modeled in ANSYS using nonlinear three-dimensional pipe elements. Yielding and buckling limit states are considered. The proposed technique is found to have a highly efficient convergence to the system reliability estimate compared to basic Monte Carlo simulation.

The proposed method may have a significant impact on the application of probabilistic methods to structural system design. Current design methods that use empirical safety factors to achieve conservatism and reliability may not be optimum for modern, complex systems. It is useful to develop quantitative estimates of reliability and the sensitivity information on risk factors. However, currently used methods for safety and reliability evaluation are empirical, based on experimental testing. Such an approach is too expensive for complex systems, and a model-based simulation approach is desirable. The proposed method achieves both accuracy and efficiency compared to other system reliability methods. The enumeration method is tedious and only provides approximate bounds on the system failure probability. On the other hand, basic Monte Carlo simulation is too expensive for practical problems where the probability of failure is quite low. The proposed adaptive importance sampling technique achieves the accuracy of basic Monte Carlo simulation with much fewer samples.

The Monte Carlo method has the advantage of modular implementation. That is, it does not interfere with the basic system analysis model, thus facilitating the use of commercial codes. The proposed adaptive sampling method preserves this advantage, while improving the speed of convergence. Therefore, this method can be applied to the reliability evaluation of a wide class of structures. Also, the method accommodates different types of component failures and consequences, such as brittle, ductile, and semi-ductile failures. Such flexibility and accommodation facilitates the implementation of probabilistic concepts for structural system reliability evaluation.

References

[1] Malla, R., Nash, W., and Lardner, T., "Thermal Effects on Very Large Space Structures," *Journal of Aerospace Engineering*, Vol. 1, No. 3, 1988, pp. 171–190.

[2] Malla, R., Nash, W., and Lardner, T., "Motion and Deformation of Very Large Space Structures," *AIAA Journal*, Vol. 27, No. 3, 1989, pp. 374–376.

[3] Malla, R., and Nalluri, N., "Dynamic Effects of Member Failure on the Response of Truss-Type Space Structures," *Journal of Spacecraft and Rockets*, Vol. 32, No. 3, 1995, pp. 545–551.

[4] Pai, S. S., and Chamis, C. C., "Probabilistic Structural Analysis of a Truss Typical for a Space Station," NASA TM-103277, Sept. 1990.

[5] Pai, S. S., and Chamis, C. C., "Probabilistic Progressive Buckling of Trusses," NASA TM-105162, April 1991.

[6] Malla, R., and Pai, S., "Probabilistic Response of a Truss-Type Space Structure with Joint and Member Imperfection," *Journal of Spacecraft and Rockets*, Vol. 32, No. 5, 1995, pp. 870–877.

[7] Mikulas, M. M., Jr., Bush, H. G., and Card, M. F., "Structural Stiffness, Strength and Dynamic Characteristics of Large Tetrahedral Space Truss Structures," NASA TM-X74001, 1977.

[8] Wu, K. C., and Lake, M. S., "Multicriterion Preliminary Design of a Tetrahedral Truss Platform," *Journal of Spacecraft and Rockets*, Vol. 33, No. 3, 1996, pp. 410–415.

[9] Mahadevan, S., "Physics-Based Reliability Models," *Reliability-Based Mechanical Design*, edited by T. A. Cruse, Marcel Dekker, New York, 1997, pp. 197–232.

[10] Mahadevan, S., "System Reliability Analysis," *Reliability-Based Mechanical Design,* edited by T. A. Cruse, Marcel Dekker, New York, 1997, pp. 233–264.

[11] Haldar, A., and Mahadevan, S., *Probability, Reliability and Statistical Methods for Engineering Design*, Wiley, New York, 2000.

[12] Breitung, K., "Asymptotic Approximations for Multinormal Integrals," *Journal of Engineering Mechanics*, Vol. 110, No. 3, 1984, pp. 357–366.

[13] Tvedt, L., "Distribution of Quadratic Forms in Normal Space: Application to Structural Reliability," *Journal of Engineering Mechanics*, Vol. 116, No. 6, 1990, pp. 1183–1197.

[14] Gollwitzer, S., and Rackwitz, R., "An Efficient Numerical Solution to the Multinormal Integral," *Probabilistic Engineering Mechanics*, Vol. 3, No. 2, 1988, pp. 98–101.

[15] Cornell, C. A., "Bounds on the Reliability of Structural Systems," *Journal of Structural Engineering*, Vol. 93, No. ST1, 1967, pp. 171–200.

[16] Ditlevsen, O., "Narrow Reliability Bounds for Structural Systems," *Journal of Structural Mechanics*, Vol. 3, 1979, pp. 455–472.

[17] Xiao, Q., and Mahadevan, S., "Second-Order Upper Bounds on Probability of Intersection of Failure Events," *Journal of Engineering Mechanics*, Vol. 120, No. 3, 1994, pp. 670–675.

[18] Xiao, Q., and Mahadevan, S., "Fast Failure Mode Identification for Ductile Structural System Reliability," *Structural Safety*, Vol. 13, No. 4, 1994, pp. 207–226.

[19] Harbitz, A., "An Efficient Sampling Method for Probability of Failure Calculation," *Structural Safety*, Vol. 3, No. 1, 1986, pp. 109–115.

[20] Melchers, R. E., "Importance Sampling in Structural Systems," *Structural Safety*, Vol. 6, No. 1, 1989, pp. 3–10.

[21] Mori, Y., and Ellingwood, B. R., "Time-Dependent System Reliability Analysis by Adaptive Importance Sampling," *Structural Safety*, Vol. 12, No. 1, 1993, pp. 59–73.

[22] Karamchandani, A., Bjerager, P., and Cornell, C. A., "Adaptive Importance Sampling," *Proceedings of ICOSSAR 1989*, American Society of Civil Engineers, Reston, VA, 1989, pp. 855–862.

[23] Mahadevan, S., and Dey, A., "Adaptive Monte Carlo Simulation for Time-Dependent Reliability Analysis of Brittle Structures," *AIAA Journal*, Vol. 35, No. 2, 1997, pp. 321–326.

[24] Wu, Y.-T., "An Adaptive Importance Sampling Method for Structural System Reliability Analysis," *Reliability Technology AD-Vol. 28*, edited by T. A. Cruse, American Society of Mechanical Engineers, New York, 1992, pp. 217–232.

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